PERMITTIVITY CHARACTERIZATION FROM TRANSMISSION-LINE MEASUREMENT

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Abstract - **We analyze three accurate broadband techniques for measuring the complex permittivity of dielectric substrates using coplanar waveguide transmission-line measurements and demonstrate good agreement with single-frequency cavity measurements.**

INTRODUCTION

This paper examines three methods, two of them new, for determining the complex permittivity of dielectric substrates using coplanar waveguide (CPW) transmission-line measurements. We obtain accurate permittivity results for lanthanum aluminate ($LaAlO_3$), gallium arsenide (GaAs), and fused silica (SiO_2) over a broad frequency range (45 MHz - 40 GHz). We verify the accuracy of the permittivity measurements at a single-frequency with the Kent resonator method [1].

EQUIVALENT IMPEDANCE METHOD

The first method we investigated, the equivalent impedance method, uses two sets of CPW with identical conductor geometries fabricated on different substrates. The first set of CPW transmission lines, the reference CPW, are fabricated on a sapphire substrate, whose loss is low and permittivity is nearly constant with frequency. We measured the propagation constant γ_r of the reference CPW using the multiline TRL calibration technique [2]; we used the methods of [3] and [4] to find C_{r0}, the frequency independent capacitance per unit length of the reference CPW. Due to the low loss of the reference CPW substrate (sapphire) its conductance G_r per unit length is negligible compared with ωC_r [4].

The second set of lines, the test lines, are fabricated on a substrate whose permittivity is to be determined. As with the reference CPW, we measured the propagation constant γ_t of the test CPW, with a multiline TRL calibration.

The ratio of the two propagation constants is

$$\frac{\gamma_t(\omega)}{\gamma_r(\omega)} = \sqrt{\frac{(R_t + j\omega L_t)(G_t + j\omega C_t)}{(R_r + j\omega L_r)(G_r + j\omega C_r)}} \approx \sqrt{\frac{(R_t + j\omega L_t)(G_t + j\omega C_t)}{(R_r + j\omega L_r)(j\omega C_{r0})}}, \quad \textbf{(1)}$$

where R, L, G, and C are the frequency dependent equivalent circuit parameters per unit length of line and the subscripts t and r denote the test and reference CPW.

The equivalent impedance method assumes that $R_r = R_t$ and $L_r = L_t$, reasonable when the metal conductors are identical. Then (1) reduces to

$$\frac{\gamma_t(\omega)}{\gamma_r(\omega)} \approx \sqrt{\frac{G_t + j\omega C_t}{j\omega C_{r0}}}, \quad \textbf{(2)}$$

which allows us to find G_t and C_t from measurements of γ_t and γ_r.

We used the quasi-TEM model of [5] to relate the relative permittivity of the test substrate to the capacitance and conductance per unit length of the CPW through the equations

$$\epsilon_t = \frac{C_t}{2\epsilon_0 F_{low}} - \frac{F_{high}}{F_{low}} \tag{3}$$

and

$$\tan\delta_t = \frac{G_t}{2\omega\epsilon_0\epsilon_t F_{low}}, \tag{4}$$

where ϵ_t and $\tan\delta_t$ are the relative permittivity and loss tangent of the test substrate. The variables F_{low} and F_{high} are constant, functions only of the CPW metal conductor geometry. Both are terms of a Schwartz-Christoffel conformal mapping that is used to determine the capacitance and conductance of a CPW line [5].

Figures 1 and 2 show relative permittivity and loss tangent for a semi-insulating GaAs substrate measured by the equivalent impedance method in dashed lines. We determined independently the complex permittivity of the semi-insulating GaAs substrate near 9 GHz by placing an unpatterned substrate in a Kent resonator [1]. According to Ref. [6], typical uncertainties for the Kent resonator technique are $\Delta\epsilon_r = \pm\ 0.2\%$ and $\Delta\tan\delta = \pm 5 \times 10^{-5}$ [6]. While the relative permittivity measured by the equivalent impedance method in Figure 1 agrees well with the Kent resonator measurement, at low frequencies the measured relative permittivity decreases unexpectedly. Figure 2 shows that the method does not measure the loss tangent accurately.

We attribute the errors to the differences in the thickness of the metal conductors on the two samples, which violates the approximation that the resistance and inductance per unit length of line are equivalent on the reference and test CPW.

CORRECTED EQUIVALENT IMPEDANCE METHOD

We first tried to use directly the quasi-TEM CPW model of Heinrich [5] to correct for the errors due to the differences in test and reference CPW metal thicknesses. Instead of neglecting these differences, as in the equivalent impedance method, we calculated the frequency dependent resistances and inductances of the two wafers from the metal conductivities, which we determined from measurements of the dc resistance, and the metal geometries. However, when we substituted the calculated values into (1) to determine C_t and G_t the errors were significant.

While the model of Ref. [5] does not predict the resistances and inductances accurately enough to find C_t and G_t, it accurately determines the *differences* between the test and reference resistances and inductances. So we measured the resistance R_r and inductance L_r of the reference CPW with the method of [3] and [4] and approximated R_t and L_t by $R_r + \Delta R$ and $L_r + \Delta L$, where ΔR and ΔL are the calculated differences. We used

$$\frac{\gamma_t(\omega)}{\gamma_r(\omega)} \approx \sqrt{\frac{[(R_r+\Delta R)+j\omega(L_r+\Delta L)](G_t+j\omega C_t)}{(R_r+j\omega L_r)(j\omega C_{r0})}} \tag{5}$$

to estimate C_t and G_t.

Figures 1 and 2 show the results of this new method in solid lines: it removes most of the errors of the equivalent impedance method even though the typical values of $|\Delta R|/R_r$ and $|\Delta L|/L_r$ are on the order of 1 and 0.01 respectively. Figure 3 compares the relative permittivity of GaAs, SiO_2 and $LaAlO_3$ substrates measured by the corrected equivalent impedance method: here the values of ΔR and ΔL are negligible. Figure 4 shows the loss tangent results for the SiO_2 substrate, which was representative of the loss tangent measurements for GaAs and $LaAlO_3$ substrates

CALIBRATION COMPARISON METHOD

We also developed and examined a new method based on the calibration comparison technique [7], which does not require electromagnetic modeling or characterization of the CPW conductor metals. Reference [8] shows that the calibration comparison technique measures the characteristic impedance Z_0 much more accurately than conventional methods based on S-parameters measurements.

As in the other two methods, we first measure the propagation constants γ_r and γ_t of the reference and test CPW with multiline TRL calibrations [2]. Using the reference CPW to provide the impedance reference, we apply the method of [7] to directly determine the characteristic impedance Z_{0t} of the test CPW. We calculate G_t and C_t from

$$G_t+j\omega C_t=\frac{\gamma_t}{Z_{0t}} \quad \textbf{(6)}$$

and the permittivity and loss tangent of the test substrate from (3) and (4).

In all cases Figures 1-4 show fair agreement between the calibration comparison method and the Kent resonator technique.

CONCLUSION

We have developed and compared three techniques for measuring the complex permittivity of dielectric substrates: the equivalent impedance method, corrected equivalent impedance method, and calibration comparison method. We find that when the conductor metal thickness on the reference and test CPW are nearly the same all three methods show good agreement. However, in the case of dissimilar conductor metal thicknesses, only the corrected equivalent impedance method and calibration comparison method agree with the Kent resonator measurements near 10 GHz and the expected behavior of low-loss dielectrics over the entire frequency range. Of the two, the calibration comparison method is simpler since it requires no electromagnetic modeling or knowledge of the CPW conductor metal geometry, but its random uncertainty appear to be larger than those of the corrected equivalent impedance method.

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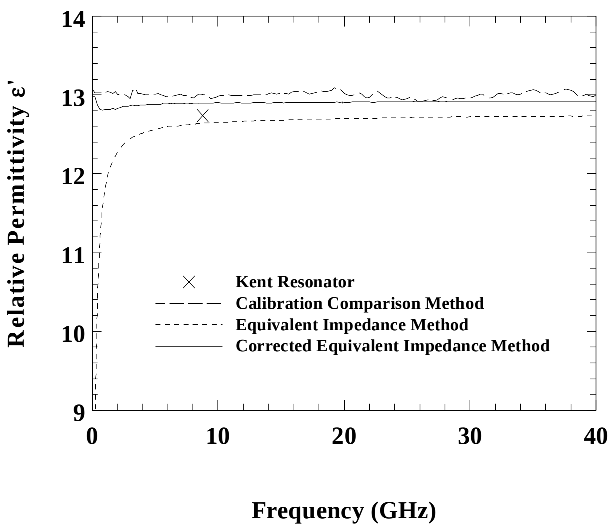

Fig. 1. The measured relative permittivity of a semi-insulating gallium arsenide substrate The metal thickness of the sapphire reference CPW is 5.71 µm, while the metal thickness of the gallium arsenide test CPW is 2.99 µm.

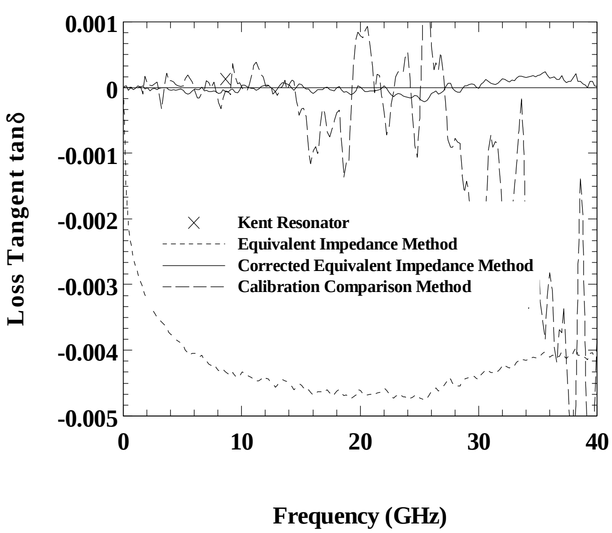

Fig. 2. The loss tangent of the semi-insulating gallium arsenide substrate of Fig. 1. for different reference and test CPW metal thicknesses.

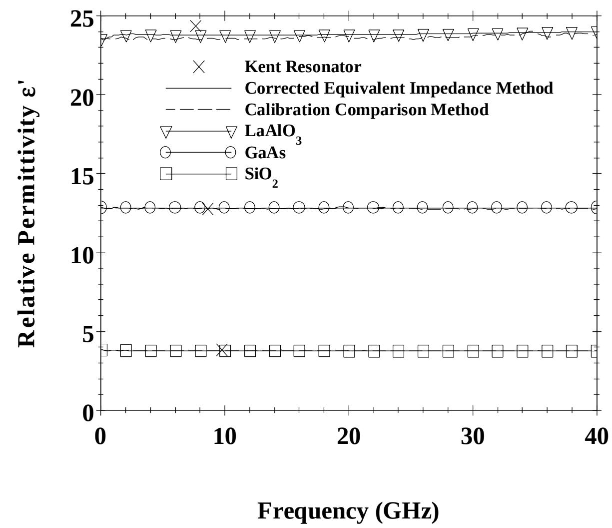

Fig 3. The measured relative permittivity of lanthanum aluminate, gallium arsenide, and fused silica substrates. The metal thickness of the sapphire reference CPW is 5.71 µm, while the metal thicknesses of the three test CPW vary in the range 4.46 - 5.52 µm.

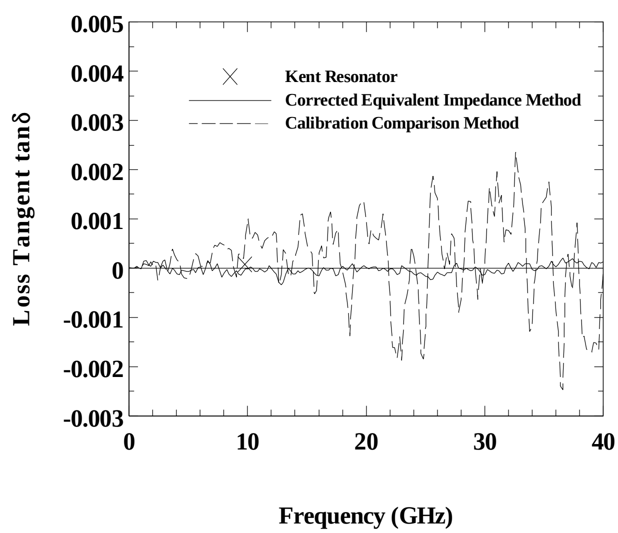

Fig. 4. The loss tangent of the fused silica substrate of Fig. 3 for nearly equal reference and test CPW metal thicknesses.